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# Diagnostics of systematic errors in angle measurements

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**Abstract**

Geodetic observations are influenced by a total error, which comprises of the systematic and random components. While, random errors are avoidable, the systematic ones should be eliminated or minimalized by the technology of measurement or by the observation corrections. Hence, the systematic errors are necessary to diagnose by the instrument calibration. This paper describes the process of diagnostics of the systematic errors, which influence angle measurements realized by total station. The diagnostic consists on the finding and specification of the particular components of instrument errors to eliminate them from the angle measurements.


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*Keywords:* Systematic errors; Total station; Gauss-Markov model, Progressive means.

## 1. Introduction

Angular and distance measurements belong to the basic geodetic terrestrial methods to collect data by contact or contactless total stations and scanning technologies. To maintain and ensure the high level of accuracy of the geodetic output data, it is important to control and test used instruments and other supported equipment to avoid or reduce the effect of survey errors. Basically, we can divide these errors into three groups: instrumental, personal and natural errors. Personal and natural errors influence the observation process by both of systematic and random effect, instrumental errors seems to have systematic character and most of them are possible to eliminate on two faces. While the quality of the new geodetic instruments is guaranteed by a producer the older one are recommended to determine the magnitude of instrumental errors and consequently to correct them. There are manufactured and other

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various professional services for providing calibration of geodetic instruments by electronic calibration procedures, but besides them we can use also laboratory tests, which are useful to perform them between full calibration to make sure the instrument accuracy.

## 2. Angular errors

The existence of angular errors of a total station depends on geometrical relation between instrumental axes and therefore they are known as axial errors:

- *Horizontal error* can be found if the trunnion axis is not perpendicular to the vertical axis instrument has,
- *Telescope eccentricity* arises if the line of sight is not in the vertical axis,
- *Collimation error* is error of line of sight axis,
- *Horizontal axis offset* means offset between horizontal and plumb axis,
- *Line of sight offset* means offset between horizontal axis and telescope axis,
- *Vertical index error* arises if the vertical axis is not plumb.

The very useful and simple method to identify these axial errors is horizontal and angular observation on geodetic marks stabilized on vertical line. The process of error diagnostics consist in two face observation both of horizontal and vertical angles. Because of the fact, that the above mentioned instrumental errors appear in the angles differences of two face observations, the mathematical model of data adjustment is based on two equations:

$$\psi_I - \psi_{II} = \frac{2e}{s} + \frac{2k}{\sin z} + \frac{2w}{tgz} + \Delta_\psi \tag{1}$$

$$z_I - z_{II} = 2i + \frac{p}{s}\cos 2 + \frac{2q}{s}\sin z + \Delta_z \tag{2}$$

| | |
|---|---|
| $w$ | is horizontal error |
| e | is telescope eccentricity |
| $k$ | is collimation error |
| $i$ | is vertical index error |
| $p$ | is horizontal axis offset |
| $q$ | is line of sight offset |
| $\Delta_\psi$ | is noise of the horizontal angles |
| $\Delta_z$ | is noise of the vertical angles |

Both of equations (1) and (2) can be solved by Least Square Method, which is applied into Gauss-Markov model explained in matrix form:

$$\mathbf{v} = \mathbf{Adx} - \mathbf{l} \tag{3}$$

$\mathbf{v}$ is vector of residuals, $\mathbf{A}$ is matrix of coeffitients, $\mathbf{dx}$ is vector of unknown parameters and $\mathbf{l}$ is vector of observations.

## 3. Data adjustment

Random parameters called random noise with systematic influence represents the total error of mathematical model (1) and (2), which can be simplified into equation:

$$\varepsilon = c + \Delta \tag{4}$$

where the total error is real effect and is equated by the following equations:

$$\varepsilon_{\psi i} = 200 - (\psi_{Ii} - \psi_{IIi}), \tag{5}$$

$$\varepsilon_{zi} = 400 - (z_{Ii} - z_{IIi}). \tag{6}$$

The a-priori accuracy of angular observations can be estimated by the application of Law of Propagation, where the appropriate variances are calculated as follows:

$$\sigma_\varepsilon = \sigma_c + \sigma_\Delta, \tag{7}$$

$$\frac{\sum \varepsilon^2}{n} = \frac{\sum c^2}{n-1} + \frac{\sum \Delta^2}{n-1}. \tag{8}$$

The random variance represents the random noise, which is free of the constant systematic effect, but the variable systematic influence is supposing. The particular standard deviations estimated from the angular observations realized by the total station are displayed in table 1. As we can see in table (1), the most part of total error is composed by the random effect with variable systematic influence.

Table 1. A-priori standard deviations.

| Standard deviation of | Horizontal angles | Vertical angles |
|---|---|---|
| total errors | 0,00225 | 0,00224 |
| random errors | 0,00220 | 0,00217 |
| systematic errors | 0,00005 | 0,00007 |

## 4. Statistical diagnostics of systematic errors

Systematic effect of angular observations was diagnosed by the dynamical criteria. For this purpose the progressive sum of errors was solved as follows:

$$x_i = \varepsilon_i + \varepsilon_{i-1} + \ldots \varepsilon_1 \tag{9}$$

the “sliding” means were calculated:

$$x_i = \frac{1}{3}(\varepsilon_i + \varepsilon_{i-1} + \varepsilon_{i-2})$$

$$x_{i+1} = \frac{1}{3}(\varepsilon_{i+1} + \varepsilon_i + \varepsilon_{i-1}) \tag{10}$$

and the progressive means were adjusted as follows:

$$x_i = \frac{1}{i}\sum_{k=1}^{i} \varepsilon_i . \tag{11}$$

The graphical trend of these estimated progressive criteria gives notice on the inherence of systematic errors. Ascending or descending trend shows the possibility of systematic errors existence in observed data. The estimated progressive criteria are displayed in fig. 1 for horizontal angles and in fig. 2 for vertical angles.

## 5. Conclusions

Geodetic praxis provides a lot of diagnostic techniques to find the systematic errors in surveys instruments. One of very useful method is above mentioned laboratory testing the horizontal and vertical angular measurements, which comprises the estimation of the a-priori accuracy, which bring the ratio between systematic and random effect in total error of observations. If the inherence of systematic errors is confirmed by the statistical progressive techniques, the mathematical Gauss-Markov model can be used to estimate the particular systematic parameters, which represents the axial errors of instrument.

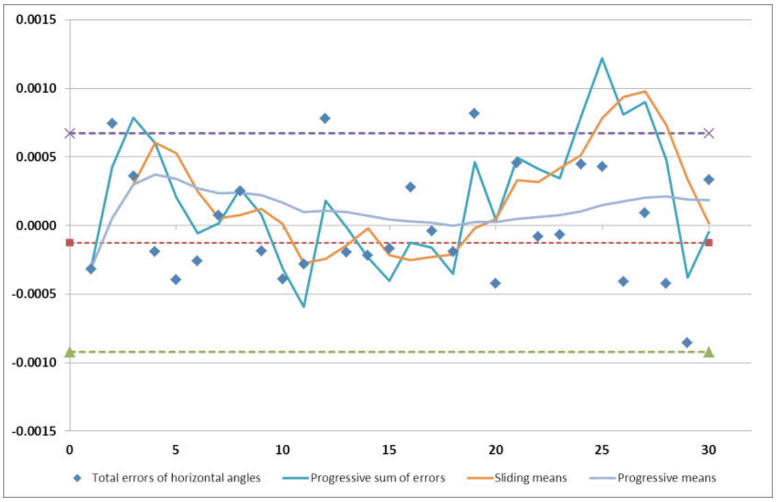


Fig. 1 Progressive criteria of systematic errors for horizontal angles.

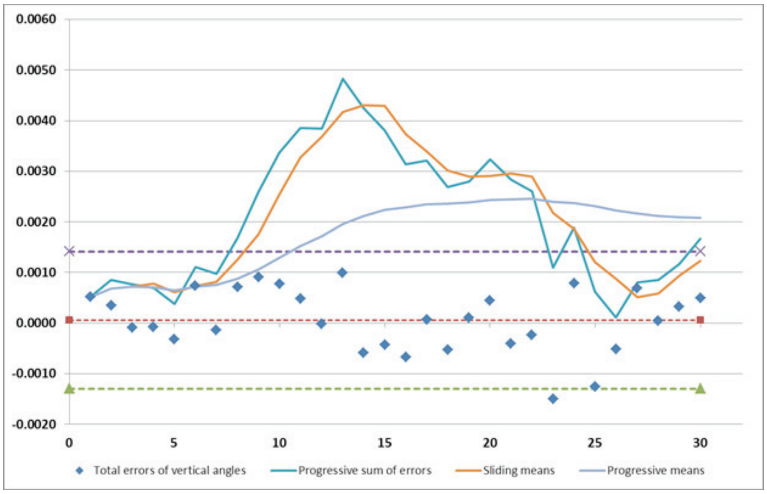


Fig. 2 Progressive criteria of systematic errors for vertical angles.

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